

The low-mass σ -meson: Is it an eyewitness of confinement?

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Abstract

In the framework of the dispersion relation N/D -method we restore the low-energy $\pi\pi$ ($IJ^{PC} = 00^{++}$)-wave amplitude sewing it with the previously obtained K -matrix solution for the region 450–1900 MeV [V.V. Anisovich, Yu.D. Prokoshkin and A.V. Sarantsev, Phys. Lett. **B389**, 388 (1996)]. The restored N/D -amplitude has a pole on the second sheet of the complex- s plane, near the $\pi\pi$ threshold. We discuss the hypothesis that this low-mass pole, or the low-mass σ -meson, corresponds to the dynamical state related to the confinement forces, that is the eyewitness of confinement.

The experimental data on meson spectra accumulated by Crystal Barrel Collaboration [1], GAMS [2] and BNL [3] groups provided a good basis for the study of ($IJ^{PC} = 00^{++}$)-wave, and the combined K -matrix analysis of the reactions $\pi\pi \rightarrow \pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$, $\pi\pi\pi\pi$ was performed over the mass range 450–1900 MeV [4]. Then the K -matrix analysis has been expanded to the waves $\frac{1}{2}0^+$ [5] and 10^+ [6], thus making it possible to establish the $q\bar{q}$ systematics of scalars for $1^3P_0q\bar{q}$ and $2^3P_0q\bar{q}$ multiplets.

The K -matrix amplitude analysis is a good instrument to perform the $q\bar{q}$ nonet classification of mesons in terms of "bare states", namely, the states without inclusion of the decay channels (detailed discussion can be found in Ref. [7]). The decay processes cause a strong mixing of scalar states, for in the transitions $(q\bar{q})_1 \rightarrow \text{real mesons} \rightarrow (q\bar{q})_2$ the orthogonality of the coordinate wave functions does not work. The other important effect caused by the decay is accumulation of widths of neighbouring resonances by one of them: as a result, a broad scalar/isoscalar state appears at the mass region 1200–1600 MeV.

According to Refs. [5, 6], the lightest scalar multiplet $1^3P_0q\bar{q}$ of bare states is located in the region 950–1200 MeV. After the mixing originated from the decay processes, the bare states of the $1^3P_0q\bar{q}$ nonet are transformed into a set of resonances: $f_0(980)$, $a_0(980)$, $f_0(1300)$, $K_0(1000)$ (or $K_0(1400)$) which are the descendants of the lightest pure $q\bar{q}$ states. The scalar/isoscalar resonances $f_0(1500)$ and $f_0(1750)$ are descendants of the bare $2^3P_0q\bar{q}$ states. At the same region there is a broad resonance, with the mass about 1200–1600 MeV, which is the descendant of the lightest scalar glueball: the gluonium and scalar/isoscalar $q\bar{q}$ states are strongly mixed (according to the rules of $1/N$ expansion [8], the transition *gluonium* $\rightarrow q\bar{q}$ is not suppressed), and the $q\bar{q}$ component in the broad state reaches 50%, while the scalars $f_0(1300)$ and $f_0(1500)$ have

considerable admixtures of gluonium components. In $f_0(980)$ and $f_0(1750)$, the $s\bar{s}$ component is predominant.

An important result of Refs. [4, 6] is that the K -matrix 00^{++} -amplitude has no pole singularities in the region 500–800 MeV. The $\pi\pi$ -scattering phase δ_0^0 increases smoothly in this energy region reaching 90° at 800–900 MeV. A straightforward explanation of such behaviour of δ_0^0 might be the presence of a broad resonance, with mass about 600–900 MeV and width $\Gamma \sim 500$ MeV [9, 10, 11, 12]. However, according to the K -matrix solution [4, 6], the 00^{++} -amplitude does not contain pole singularities on the second sheet of the complex- $M_{\pi\pi}$ plane inside the interval $450 \leq \text{Re } M_{\pi\pi} \leq 900$ MeV: the K -matrix amplitude has a low-mass pole only, which is located on the second sheet either near the $\pi\pi$ threshold or even below it. In Refs. [4, 6], the presence of this pole was not emphasized, for the left-hand cut, which plays an important role in the partial amplitude, was taken into account in indirect way only. A proper way for the description of the low-mass amplitude must be using of the dispersion relation representation.

In this paper the dispersion relation N/D -amplitude is reconstructed for the $\pi\pi$ scattering in the region $M_{\pi\pi} < 1000$ MeV, and this amplitude is sewed with the K -matrix solution of Refs. [4, 6]. In the next Section, the dispersion relation amplitude is found using the method developed in Ref. [13]: the N/D -amplitude provides a good description of δ_0^0 up to 900 MeV, thus including the region $\delta_0^0 \sim 90^\circ$. At the same time this amplitude does not contain a pole in the region 500–900 MeV; instead, the pole is located near the $\pi\pi$ threshold.

We suggest that the low-mass pole in the scalar/isoscalar wave is related to a fundamental phenomenon at large distances (in hadronic scale). In Section 2 we argue that the low-mass pole corresponds to a white composite particle which is inherent to subprocesses responsible for the colour confinement forces.

1 Dispersion relation N/D -solution for the $\pi\pi$ -scattering amplitude below 900 MeV

The pion-pion scattering partial amplitude being a function of the invariant energy squared, $s = M_{\pi\pi}^2$, can be represented as a ratio $N(s)/D(s)$, where $N(s)$ has left-hand cut, which is due to the "forces" (the interactions due to t - and u -channel exchanges), while the function $D(s)$ is determined by the rescatterings in the s -channel. $D(s)$ is given by the dispersion integral along the right-hand cut in the complex- s plane:

$$A(s) = \frac{N(s)}{D(s)}, \quad D(s) = 1 - \int_{4\mu_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\rho(s')N(s')}{s' - s + i0}. \quad (1)$$

Here $\rho(s)$ is the invariant $\pi\pi$ phase space, $\rho(s) = (16\pi)^{-1} \sqrt{(s - 4\mu_\pi^2)/s}$. It is supposed in (1) that $D(s) \rightarrow 1$ with $s \rightarrow \infty$ and CDD-poles are absent (a detailed presentation of the N/D -method can be found in [14]).

The N -function can be written as an integral along the left-hand cut as follows:

$$N(s) = \int_{-\infty}^{s_L} \frac{ds'}{\pi} \frac{L(s')}{s' - s}, \quad (2)$$

where the value s_L marks the beginning of the left-hand cut. For example, for the one-meson exchange diagram $g^2/(m^2 - t)$, the left-hand cut starts at $s_L = 4\mu_\pi^2 - m^2$, and the N -function in this point has a logarithmic singularity; for the two-pion exchange, $s_L = 0$.

Below we work with the amplitude $a(s)$, which is defined as follows:

$$a(s) = \frac{N(s)}{8\pi\sqrt{s} \left(1 - P \int_{4\mu_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\rho(s')N(s')}{s' - s} \right)}. \quad (3)$$

The amplitude $a(s)$ is related to the scattering phase shift: $a(s)\sqrt{s/4 - \mu_\pi^2} = \tan\delta_0^0$. In Eq. (3) the threshold singularity is singled out explicitly, so the function $a(s)$ contains left-hand cut only as well as the poles corresponding to zeros of the denominator of the right-hand side (3): $1 = P \int_{4\mu_\pi^2}^{\infty} (ds'/\pi) \cdot \rho(s')N(s')/(s' - s)$. The pole of $a(s)$ at $s > 4\mu_\pi^2$ corresponds to the phase shift value $\delta_0^0 = 90^\circ$. The phase of the $\pi\pi$ scattering reaches the value $\delta_0^0 = 90^\circ$ at $\sqrt{s} = M_{90} \simeq 850$ MeV. Because of that, the amplitude $a(s)$ may be represented in the form:

$$a(s) = \int_{-\infty}^{s_L} \frac{ds'}{\pi} \frac{\alpha(s')}{s' - s} + \frac{C}{s - M_{90}^2} + D. \quad (4)$$

For the reconstruction of the low-mass amplitude, the parameters D, C, M_{90} and $\alpha(s)$ have been determined by fitting to the experimental data. In the fit we have used the method, which has been approved in the analysis of the low-energy nucleon-nucleon amplitudes [13]. Namely, the integral in the right-hand side of (4) has been replaced by the sum

$$\int_{-\infty}^{s_L} \frac{ds'}{\pi} \frac{\alpha(s')}{s' - s} \rightarrow \sum_n \frac{\alpha_n}{s_n - s} \quad (5)$$

with $-\infty < s_n \leq s_L$.

The description of data within the N/D -solution, which uses six terms in the sum (5), is demonstrated on Fig. 1a. Parameters of the solution are given at Table 1. The scattering length in this solution is equal to $a_0^0 = 0.22 \mu_\pi^{-1}$, the Adler zero is at $s = 0.12 \mu_\pi^2$. The N/D -amplitude is sewed with the K -matrix amplitude of Refs. [4, 6], and figure 1b demonstrates the level of the coincidence of the amplitudes $a(s)$ for both solutions (the values of $a(s)$ which correspond to the K -matrix amplitude are shown with error bars determined in [4, 6]).

The dispersion relation solution has a correct analytic structure at the region $|s| < 1$ GeV². The amplitude has no poles on the first sheet of the complex- s plane; the left-hand cut of the N -function after the replacement given by Eq. (5) is transformed into a set of poles on the negative piece of the real s -axis: six poles of the amplitude (at $s/\mu_\pi^2 =$

$-5.2, -9.6, -10.4, -31.6, -36.0, -40.0$) represent the left-hand singularity of $N(s)$. On the second sheet (under the $\pi\pi$ -cut) the amplitude has two poles: at $s \simeq (4 - i14)\mu_\pi^2$ and $s \simeq (70 - i34)\mu_\pi^2$ (see Fig. 2). The second pole, at $s = (70 - i34)\mu_\pi^2$, is located beyond the region under consideration, $|s| < 1 \text{ GeV}^2$ (nevertheless, let us stress that the K -matrix amplitude [4, 6] has a set of poles just in the region of the second pole of the N/D -amplitude). The pole near threshold, at

$$s \simeq (4 - i14)\mu_\pi^2, \quad (6)$$

is what we discuss. The N/D -amplitude has no poles at $\text{Re}\sqrt{s} \sim 600 - 900 \text{ MeV}$ despite the phase shift δ_0^0 reaches 90° here.

The data do not fix the N/D -amplitude rigidly. The position of the low-mass pole can be easily varied in the region $\text{Re } s \sim (0 - 4)\mu_\pi^2$, and there are simultaneous variations of the scattering length in the interval $a_0^0 \sim (0.21 - 0.28)\mu_\mu^{-1}$ and Adler zero at $s \sim (0 - 1)\mu_\pi^2$.

The problem of the low-mass σ -meson was discussed previously. In the approaches which take into account the left-hand cut, the following positions of the pole singularities were found:

- (i) dispersion relation approach, $s \simeq (0.2 - i22.5)\mu_\pi^2$ [15],
- (ii) meson exchange models, $s \simeq (3.0 - i17.8)\mu_\pi^2$ [16], $s \simeq (0.5 - i13.2)\mu_\pi^2$ [17],
 $s \simeq (2.9 - i11.8)\mu_\pi^2$ [18],
- (iii) linear σ -model, $s \simeq (2.0 - i15.5)\mu_\pi^2$ [19],

which are in an agreement with our result.

However in Refs. [20, 21], the pole singularities were obtained in the region of higher masses, at $\text{Re } s \sim (7 - 10)\mu_\pi^2$.

2 Low-mass pole as the eyewitness of confinement

We believe that the existence of the low-mass pole in the 00^{++} -amplitude is not an occasional event. The creation of a composite scalar/isoscalar particle with small mass should be associated with certain fundamental phenomenon at separations, which are large in hadronic scale: such a phenomenon may be the confinement.

The confinement interaction is modelled by the scalar potential in colour octet state, $V_8(r)$; the potential increases infinitely at large distances, $V_8(r) \sim r$ at $r \rightarrow \infty$. The formation of the confinement potential is promptly related to the creation of a set of $q\bar{q}$ -pairs, or a $q\bar{q}$ -chain. The examples are provided by the multihadron production at $\mu^+\mu^-$ -annihilation (the transition $\gamma^* \rightarrow q\bar{q}$) or highly excited meson decay (transition $M^* \rightarrow q\bar{q}$, see Fig. 3a). In the decay process of Fig. 3a, the flying away colour quarks produce a chain of the $q\bar{q}$ -pairs due to which the colour of the upper quark flees to the bottom quark. The process of Fig. 3a results in cutting the self-energy diagram of Fig. 3b. The interaction block inside the quark loop is responsible for the formation of the colour potential $V_8(r)$; this block is shown separately in Fig. 3c.

The increase of $V_8(r) \sim r$ at large r means the existence of a strong singularity in the momentum representation at small $|\vec{q}|$: $V_8(\vec{q}^2) \sim 1/\vec{q}^4$. Using notations of Fig. 3c, one has $-\vec{q}^2 = s$. So, the set of diagrams of Fig. 3c-type being responsible for the confinement forces has a strong singularity near $s = 0$.

The infinite increase of $V_8(r) \sim r$ at $r \rightarrow \infty$, which is reproduced in the singular behaviour $V_8(q^2) \sim 1/q^4$ at $q^2 \rightarrow 0$, represents an idealized picture of the confinement. In this picture

the limit $V_8(q^2) \rightarrow 1/q^4$ at $q^2 \rightarrow 0$ can be interpreted as an exchange by massless composite colour-octet particle with a coupling growing infinitely: $g^2(q^2)/q^2$ with $g^2(q^2) \sim 1/q^2$ at small q^2 . This composite colour-octet particle interacts as a whole system only at large distances: its compositeness guarantees the return to standard QCD at small r . At large distances, the strongly interacting colour-octet massless particle provides colour neutralization of the quark/gluon systems.

However, the colour screening effects (in particular, encountered in the production of white particles) cut the infinite growth of $V_8(r)$ at very large r allowing the behaviour $V_8(r) \sim r$ at $r < R_{\text{confinement}}$ only (for example, see Ref. [22]). It results in a softening of singular behaviour of $V_8(q^2)$ at $q^2 \sim 0$. Nevertheless, the s -channel pole singularities in the block of Fig. 3c-type may survive. Our point is that in this case the singularities at $s \simeq 0$ reveal themselves not only in the colour-octet state but in colour-singlet one as well, i.e. in the $\pi\pi$ -scattering amplitude.

To clarify this point, let us re-write the block of Fig. 3c in terms of the colour/flavour propagators (t'Hooft–Veneziano diagram). In Fig. 3d, the t'Hooft–Veneziano diagram which corresponds to the block of Fig. 3c is shown: solid line represents the propagation of colour and dashed line describes that of flavour. It is seen that

- (i) in the s -channel the block contains two open colour lines, $c = 3$ and $\bar{c} = 3$, so one has two colour states in the s -channel, octet and singlet: $c \otimes \bar{c} = 1 + 8$.
- (ii) the two s -channel colour amplitudes, octet and singlet ones, are not suppressed in the $1/N$ -expansion ($N = N_c = N_f$), that means that the same leading- N subgroups of the diagrams are responsible for the formation of the octet and singlet amplitudes. Therefore, the positions of the pole singularities in both amplitudes are the same in the leading- N terms. However, the next-to-leading terms (such as decay of the white state into $\pi\pi$) discharge the rigid degeneration of the octet and singlet states.

Summing up, in the leading- N terms the pole structure near $q^2 = 0$ is the same both for colour-singlet and colour-octet amplitudes. The colour-octet amplitude providing the growth of the potential $V_8(r) \sim r$ at $r \sim R_{\text{confinement}}$ can have singularities near $q^2 = 0$. This means that the colour-singlet amplitude has similar singularities. Because of that, corresponding white state can be named the eyewitness of confinement.

The existence of states, which may be called the eyewitnesses of confinement, was discussed in Ref. [23], though in Ref. [23] the scalar quartet ($f_0(980)$, $a_0(980)$) was under discussion.

3 Conclusion

We have analysed the structure of the low-mass $\pi\pi$ -amplitude in the region $M_{\pi\pi} \lesssim 900$ MeV using the dispersion relation N/D -method, which provides us with a possibility to take the left-hand singularities into consideration. The dispersion relation N/D -amplitude is sewed with that given by the K -matrix analysis performed at $M_{\pi\pi} \sim 450 - 1950$ MeV [4]. Obtained in this way the N/D -amplitude has a pole on the second sheet of the complex- s plane near the $\pi\pi$ threshold. This pole corresponds to the existence of the low-energy bound state.

On the basis of the conventional confinement model, we argue that the confinement interaction can produce low-mass pole singularities both for the colour-octet and colour-singlet states. Then the colour-singlet scalar state, which reveals itself in the low-mass $\pi\pi$ -amplitude,

may be named the eyewitness of confinement.

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Table 1: Fitting parameters for the amplitude $a(s)$, Eqs. (4), (5).

$s_n \mu_\pi^{-2}$	-9.56	-10.16	-10.76	-32	-36	-40
$\alpha_n \mu_\pi^{-1}$	2.21	2.21	2.21	0.246	0.246	0.246
$M_{90} = 6.228 \mu_\pi, \quad C = -13.64 \mu_\pi, \quad D = 0.316 \mu_\pi^{-1}$						

References

- [1] V.V. Anisovich et al. Phys. Lett. **B323** (1994) 233;
C. Amsler et al., Phys. Lett. **B333** (1994) 277; **B355** (1995) 425.
- [2] D. Alde et al., Phys. Lett. **B241** (1990) 600; Zeit. Phys. **C66** (1995) 375;
Yu. D. Prokoshkin et al., Physics-Doklady **342** (1995) 473;
F. Binon et al., Nuovo Cim. **A78** (1983) 313; **A80** (1984) 363.
- [3] S. J. Lindenbaum and R. S. Longacre, Phys. Lett. **B274** (1992) 492;
A. Etkin et al., Phys. Rev. **D25** (1982) 1786.
- [4] V. V. Anisovich, Yu. D. Prokoshkin and A. V. Sarantsev, Phys. Lett. **B389** (1996) 388.
- [5] A.V. Anisovich and A.V. Sarantsev, Phys. Lett. **B413** (1997) 137.
- [6] V. V. Anisovich, A. A. Kondashov, Yu. D. Prokoshkin, S. A. Sadovsky, and A. V. Sarantsev, "The two-pion spectra for the reaction $\pi^- p \rightarrow \pi^0 \pi^0 n$ at 38 GeV/c pion momentum and combined analysis of the GAMS, Crystal Barrel and BNL data", hep-ph/9711319, Phys. of Atomic Nuclei, in press.
- [7] A. V. Anisovich, V. V. Anisovich and A. V. Sarantsev, Zeit. Phys. **A359** (1997) 173.
- [8] G. t'Hooft, Nucl. Phys. **B72** (1974) 161;
G. Veneziano, Nucl. Phys. **B117** (1976) 519.
- [9] S.D. Protopopescu et al. Phys. Rev. **D7** (1973) 1279.
- [10] P. Estabrooks, Phys. Rev. **D19** (1979) 2678.
- [11] K.L. Au, D. Morgan and M.R. Pennington, Phys. Rev. **D35** (1987) 1633.

- [12] S. Ishida et al. *Prog. Theor. Phys.* **98** (1997) 1005.
- [13] V.V. Anisovich, M.N. Kobrinsky, D.I. Melikhov and A.V. Sarantsev, *Nucl. Phys.* **A544** (1992) 747.
- [14] G.F. Chew, *The analytic S-matrix*, NY, Benjamin, 1966.
- [15] J.L. Basdevant, C.D. Frogatt and J.L. Petersen, *Phys. Lett.* **B41** 178 (1972).
- [16] J.L. Basdevant and J. Zinn-Justin, *Phys. Rev.* **D3** (1971) 1865;
D. Iagolnitzer, J. Justin and J.B. Zuber, *Nucl. Phys.* **B60** (1973) 233.
- [17] B.S. Zou and D.V. Bugg, *Phys. Rev.* **D48** (1994) R3942; **D50** (1994) 591.
- [18] G. Janssen, B.C. Pearce, K. Holinde and J. Speth, *Phys. Rev.* **D52** (1995) 2690.
- [19] N.N. Achasov and G.N. Shestakov, *Phys. Rev.* **D49** (1994) 5779.
- [20] V.E. Markushin and M.P. Locher, "Structure of the light scalar mesons", Talk given at the Workshop on Hadron Spectroscopy, Frascati, 1999, preprint PSI-PR-99-15.
- [21] N.A. Törnquist and M. Roos, *Phys. Rev. Lett.* **76** (1996) 1575.
- [22] V.N. Gribov, Preprint LU-TP 91-7, Lund University, 1991.
- [23] F.E. Close et al. *Phys. Lett.* **B319** (1993) 291.

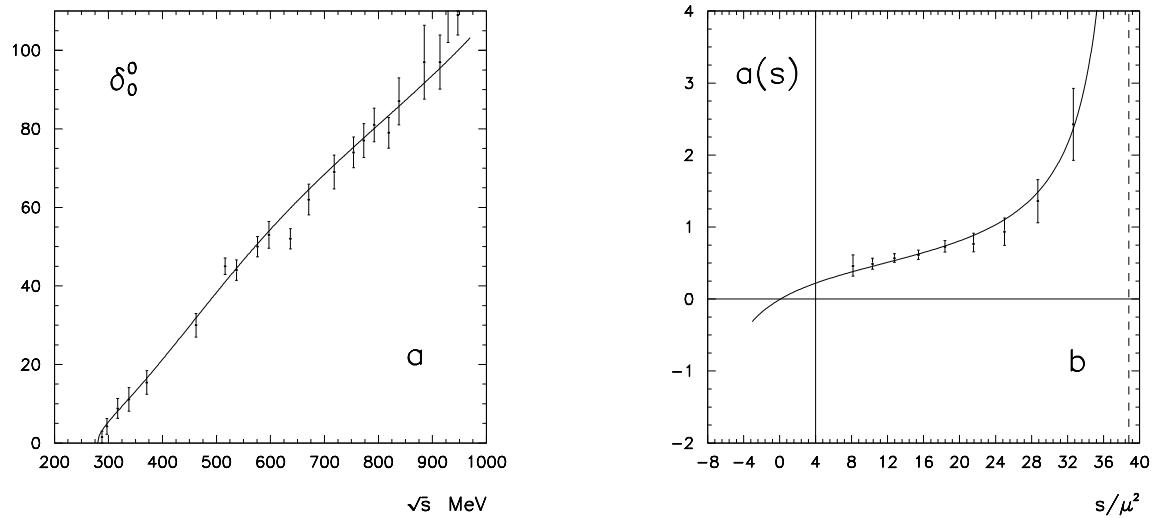


Figure 1: a) Fit to the data on δ_0^0 by using the N/D -amplitude. b) Amplitude $a(s)$ in the N/D -solution (solid curve) and the K -matrix approach [4,6] (points with error bars).

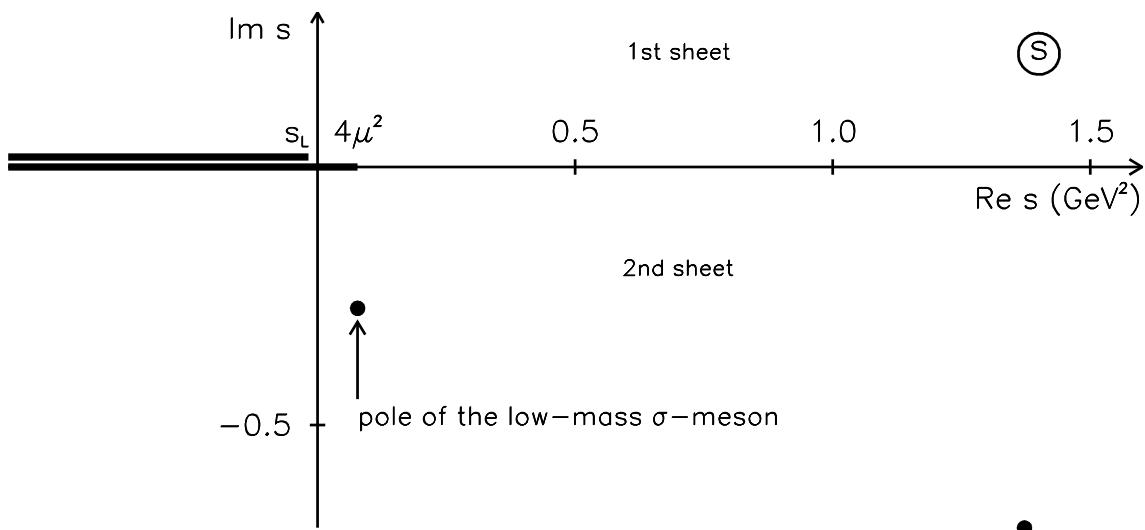


Figure 2: Complex-s plane and singularities of the N/D -amplitude

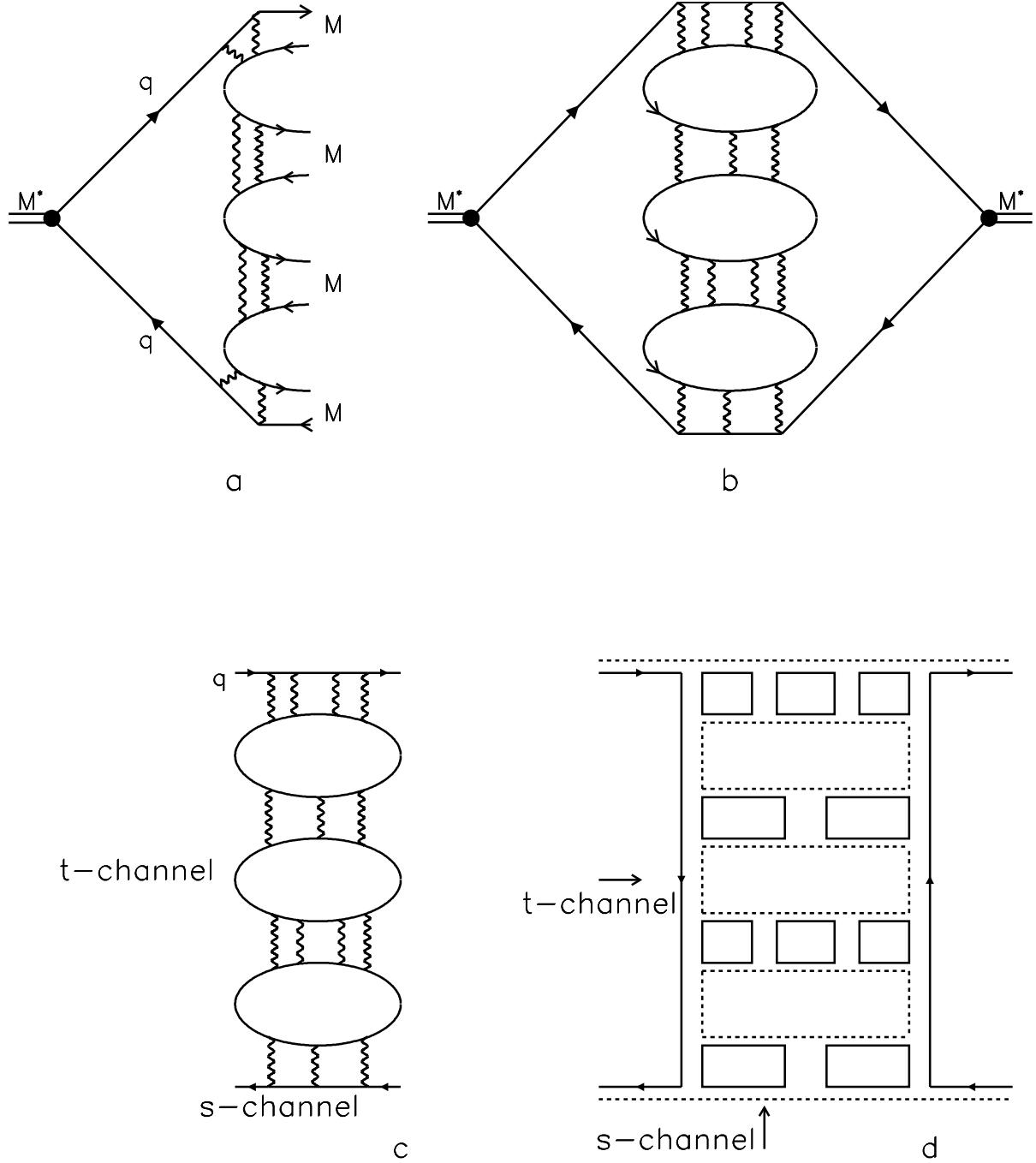


Figure 3: Diagrams for the block responsible for the confinement interaction: a) transition $M^* \rightarrow \text{hadrons}$ with the chain of quark-antiquark pairs; b) self-energy part the cutting of which gives the diagram of Fig. 3a; c) the interaction block of the self-energy diagram of Fig. 3b: a set of such diagrams yields the confinement potential $V_8(r)$; d) the t'Hooft–Veneziano diagram for the block of Fig. 3c: solid and dashed lines represent the propagation of colour and flavour, respectively.